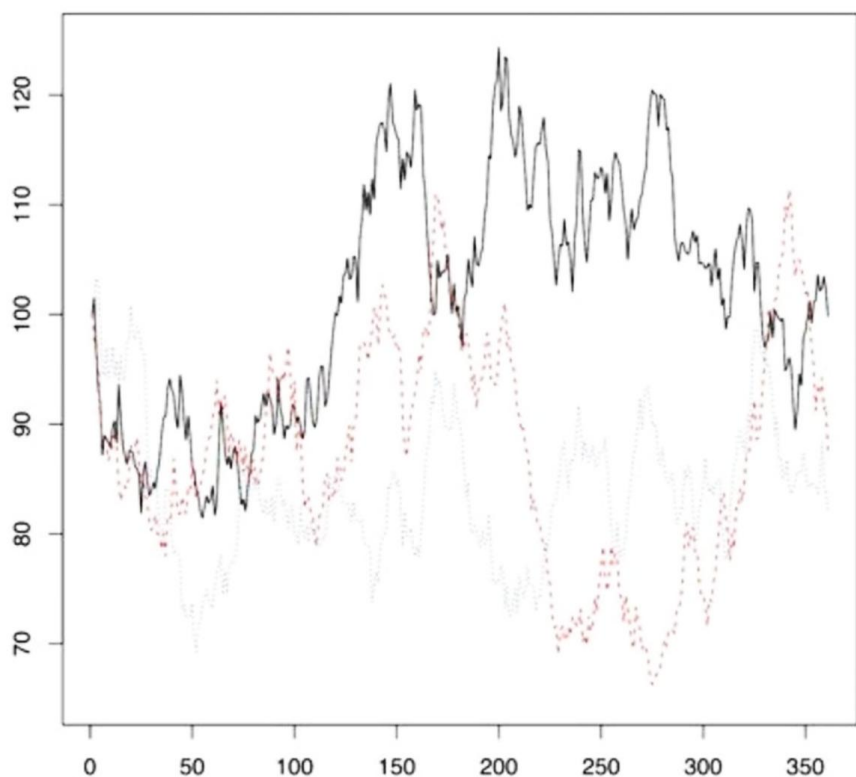


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MARIA C. MARIANI | IONUT FLORESCU

QUANTITATIVE FINANCE



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Quantitative Finance

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